

A comment on chiral restoration at finite baryon density in hyperspherical unit cells

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Prompted by recent work of Adhikari, Cohen, Ayyagari and Strother *On chiral symmetry restoration at finite density in large- N_c QCD* (Phys. Rev. C **83**, 065201 (2011)), we revisit the description of dense baryonic matter in terms of hyperspherical unit cells. We focus mainly on the interpretation of the unique energy, curvature and symmetry properties which enable such S^3 cells to describe full chiral restoration in Skyrme models and which markedly distinguish them from the flat and periodic unit cells of Skyrmin crystals. These key features clarify, in particular, why an S^3 cell interpretation as a crystal-cell model in which the specific cell geometry is without physical significance, as tentatively adopted by Adhikari et al., is insufficient. The ensuing criticism does therefore not apply to the usual interpretation of S^3 cells which we describe. We also suggest a few directions in which the latter interpretation may be developed further.

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I. INTRODUCTION AND MOTIVATION

The recent work of Adhikari et al. on chiral symmetry restoration at high baryon densities in QCD with a large number N_c of colors [1] contains a section which discusses chiral restoration in the “hypersphere approach” [2–4]. The latter describes dense matter in models of Skyrme type [5] by means of generalized, curved unit cells with the geometry of a three-dimensional sphere S^3 . This geometry was selected because the interplay of its symmetry group $SO(4)$ with the chiral $SU(2) \times SU(2) \simeq SO(4)$ symmetry of the dynamics uniquely enables such hyperspherical cells to model chiral restoration beyond a critical density [2, 3]. In fact, S^3 is the *only* unit-cell geometry in which the Skyrminion can attain its absolute energy minimum and in which the transition to a chirally restored phase can take place.

The above properties provided the original motivation for studying S^3 cells in the Skyrme model [2, 3]. At this stage no analogies with the flat unit cells of periodic Skyrminion arrays were made. Only later it was realized that the discrete “half-Skyrmion” symmetry [6], which emerges in the high-density phase of Skyrminion crystals [7], is restored together with the chiral group on S^3 as well [4]. Since several consequences of full chiral restoration on S^3 turned out to require just the half-Skyrmion symmetry, it was further argued in Ref. [4] that the emergence of the latter should be interpreted as signaling chiral restoration in the restricted setting of Skyrminion crystals. Moreover, calculations in S^3 cells proved to be far less complex than those in flat, periodic unit cells [24]. The crucial impact of the specific hypersphere properties, especially on the density-dependent multiplet structure of the Goldstone bosons and other excitations, furthermore strengthened the original view that S^3 cells provide an independent and with regard to chiral symmetry properties more complete description of dense matter.

A rather different interpretation of hyperspherical cells was recently explored in Ref. [1]. Motivated by the suggested quarkyonic dense-matter phase at large N_c [9], the bulk of Ref. [1] investigates chiral restoration con-

ditions in Skyrme models and in large- N_c QCD. This raised the question to what extent S^3 cells can be considered as faithful models for the flat, periodic unit cells of Skyrminion crystals. To address this issue, the authors of Ref. [1] adopt the position that the sole purpose of hyperspherical unit cells should be to “approximate a Skyrminion in the crystal” [25]. In addition, they assume “that the principal effect of putting a Skyrminion into a crystal is to restrict the space over which it can spread” and “that using a hypersphere to restrict the volume of the Skyrminion acts generically like other restrictions on its volume”. Below we will explain why these assumptions, which would deprive the cell geometry of its physical significance [1], are oversimplifications. Indeed, they ignore the unique energy, symmetry and curvature properties of the S^3 geometry and would not even hold for the flat unit cells of Skyrminion crystals. In addition, these premises lead the authors of Ref. [1] to the unduly pessimistic conclusions that “the special properties of the geometry ... make the (gained) intuition totally unreliable even for qualitative issues associated with chiral symmetry breaking and its possible restoration in the average sense” and that the “evidence for chiral restoration ... was an artifact of the hyperspherical geometry”.

The main purpose of the present note is to clarify that the above criticism, based on the problematic interpretation of Ref. [1], does not apply to the standard S^3 cell interpretation. To this end, we will discuss the physical significance of the S^3 geometry [26], address the problems with the premises underlying the interpretation of Ref. [1], and point out an important difference between chiral restoration “in the average sense” in flat space and curved cells. Finally, we will suggest a possible extension of the S^3 unit cell interpretation.

II. THE UNIQUE “CHIRAL” SIGNIFICANCE OF S^3 UNIT CELLS

We start by reviewing those key features of hyperspherical unit cells which first suggested their physical

significance. These properties will also help to explain what prevents the related results from being “an artifact of the (unphysical) choice of geometry”, as they would appear to be in the problematic interpretation of Ref. [1]. In fact, in models of Skyrme type [5] several essential dense-matter properties turned out to be uniquely encoded into S^3 unit cells [2, 3]. Manton demonstrated this uniqueness by considering generalized Skyrmions as topologically nontrivial maps between two Riemannian manifolds, i.e. Σ_{cell} (the unit cell space) and Σ (the target space in which the fields take values) [3]. He noted, in particular, that “the metrics on both Σ_{cell} and Σ are *essential*” and that “the energy of the Skyrmon is a measure of the geometrical distortion induced by the map”. In fact, since chiral symmetry allows only gradient interactions among the pions and since those are exceptionally sensitive to the local background curvature, one expects an enhanced impact of the Σ_{cell} metric on even qualitative dense-matter predictions.

As in the nonlinear σ model, the chiral symmetry of the Skyrme dynamics is nonlinearly realized [10] on its unbroken isospin subgroup $\text{SU}(2) \sim S^3$, i.e. $\Sigma = S^3$. (For simplicity, we assume exact chiral $\text{SU}(2) \times \text{SU}(2)$ symmetry of the dynamics and exact $\text{SU}(2)$ isospin symmetry of the vacuum.) Hence the energetically privileged role of $\Sigma_{\text{cell}} = S^3$ emerges already at this qualitative level. More quantitatively, the interplay between the Skyrmon’s energy and topology results in an absolute energy minimum given by the Bogomol’ny (or Faddeev) bound [5]. Since the Skyrmon’s energy functional is an efficient measure of the metric deformation between the unit cell Σ_{cell} and the target space Σ , the field configuration which saturates the Bogomol’ny bound should not induce any such deformation. This requires both Σ_{cell} and Σ to have the same metric. Hence S^3 is the unique unit-cell geometry in which the Skyrmon can become the metric preserving identity map [27] and thereby attain its absolute energy minimum.

In addition, S^3 is the unique space which $\text{SO}(4)$ transformations leave invariant. The Skyrmon’s “hedgehog” coupling between space and isospace links this isometry group to the chiral $\text{SU}(2) \times \text{SU}(2) \simeq \text{SO}(4)$ group which acts analogously on the internal field space $\Sigma = S^3$. This provides the key to understanding why chiral restoration (beyond the critical density and in a sense to be specified below) is possible only in S^3 unit cells. Indeed, the symmetry group G of the Hamiltonian (for static fields) in general unit cells Σ_{cell} is the product of the spatial cell symmetries G_{cell} and of the chiral group $\text{SO}(4)_{\chi} \simeq \text{SO}(3)_L \times \text{SO}(3)_R$. In the presence of a semi-classically quantized Skyrmon, the symmetry of the spectrum is reduced to the subgroup of G which leaves the Skyrmon invariant. Due to the hedgehog-type coupling $\hat{x}^i \tau^i$ in the Skyrmon solution $U(\vec{x}) = \exp[i\hat{x}^i \tau^i F(|\vec{x}|)]$ this group includes the diagonal subgroup of spatial and $\text{SO}(3)_{\text{iso}}$ isospin rotations (where the latter belong to the diagonal subgroup of $\text{SO}(4)_{\chi}$).

In the familiar flat-space example $\Sigma_{\text{cell}} = \mathbb{R}^3$ one

thus has $G_{\text{cell}} = T(\mathbb{R}^3) \times \text{SO}(3)_{\text{rot}}$ where $T(\mathbb{R}^3)$ are the translations in \mathbb{R}^3 . Since the localized, classical Skyrmon breaks translational invariance while isospin rotations can be undone by spatial rotations around the Skyrmon’s center, only the diagonal subgroup $\text{SO}(3)_{\text{grand}} = \text{diag}\{\text{SO}(3)_{\text{rot}} \times \text{SO}(3)_{\text{iso}}\}$ consisting of simultaneous spatial and isospin rotations leaves the Skyrmon invariant. In other words, chiral (as well as rotational and translational) symmetry is spontaneously broken to the so-called “grand spin” from which isospin (and rotational) symmetry can be recovered by projection [11].

We now return to the description of dense matter and search for a three-dimensional unit cell Σ_{cell} whose symmetry enables a full chiral $\text{SO}(4)$ group to leave the Skyrmon invariant. The hedgehog coupling suggests to write $G_{\text{cell}} = T(\Sigma_{\text{cell}}) \times \text{SO}(3)_{\text{rot}}$ as the product of the rotations around the Skyrmon center and the coset of “generalized translations” $T(\Sigma_{\text{cell}}) := G_{\text{cell}}/\text{SO}(3)_{\text{rot}}$ which move this center around the cell. Since $\text{SO}(3)_{\text{rot}}$ is linked to $\text{SO}(3)_{\text{iso}}$ as above, with the Skyrmon leaving only their diagonal subgroup invariant, we look for an extension of the grand-spin subgroup to $\text{SO}(4)$. This requires the full $\text{SO}(4)_{\chi}$ to take part in one factor, and the latter to be multiplied by the extension of $\text{SO}(3)_{\text{rot}}$ to an analogous $G_{\text{cell}} = \text{SO}(4)$. The unique cell with this symmetry group is $\Sigma_{\text{cell}} = S^3$. Hence the hypersphere is indeed the only cell geometry in which the invariance group of the Skyrmon can become $\text{SO}(4)$.

Two generic situations must now be distinguished. For cell radii L which are large compared to the Skyrmon’s size, the Skyrmon is localized on S^3 and therefore $T(S^3)$ is spontaneously broken. For L equal to or smaller than a critical radius, on the other hand, the Skyrmon on S^3 delocalizes completely. As discussed above, at the critical radius it becomes the energy-minimizing identity map between $\Sigma_{\text{cell}} = S^3$ and $\Sigma = S^3$. Hence no center is singled out anymore either in Σ_{cell} or in Σ , and any generalized translation can be compensated by the corresponding “translation” from the axial coset $\text{SO}(4)_{\chi}/\text{SO}(3)_{\text{iso}}$ on the field manifold. The symmetry group of the identity-map Skyrmon is therefore $\text{SO}(4)_{\chi'} = \text{diag}\{\text{SO}(4)_{\text{cell}} \times \text{SO}(4)_{\chi}\}$. After projection as above, this $\text{SO}(4)_{\chi'}$ turns into the standard chiral group which is thus indeed restored.

The above, complete chiral restoration implies that all averaged chiral order parameters disappear, as noticed in Ref. [1], and leaves crucial imprints on the fluctuation spectrum around the Skyrmon in the S^3 cell [4]. Below the critical density the excitations fall (after projection) into isospin multiplets and include a triplet of massless Goldstone pions, i.e. the telltale signature of spontaneously broken chiral symmetry. At and beyond the critical density, on the other hand, the spectrum is classified by the larger, chiral $\text{SO}(4)$ group. The former Goldstone bosons, in particular, join their three parity partners in a degenerate chiral multiplet whose mass in-

creases with the density [4], as expected from complete chiral restoration.

Several additional features of the above restoration mechanism were studied later and revealed, for example, an interesting interplay with kaon condensation [12]. For more recent work on Skyrmions in hyperspherical cells see Ref. [13]. An interesting appearance of S^3 cells in the context of holographic QCD [14] is related to instantons on S^3 [15] which generate approximate Skyrmon solutions by means of the Atiyah-Manton map [16].

III. ADDITIONAL ASPECTS OF THE S^3 CELL INTERPRETATION

The key properties discussed above provide the basis for the physical interpretation of S^3 Skyrmon cells. Nevertheless, some of their more unconventional features, including several of those which set them even qualitatively apart from flat, periodic unit cells, still await a better understanding. In the present section we suggest a few directions in which the understanding of S^3 cells may potentially be improved, and we address the problems with the interpretation of Ref. [1] in more detail.

We start by recalling that S^3 unit cells were found to describe principal features of dense matter even in chiral models which are *not* of Skyrme type. In the remarkably different Nambu Jona-Lasinio (NJL) model [17], in particular, where chiral symmetry is broken by interactions among quarks which carry intrinsic baryon number but no topology, a growing baryon density described by S^3 unit cells was shown to trigger the transition to the chirally restored phase as well [18]. Simultaneously, at a critical density consistent with standard values, the previously massless and tightly bound Goldstone pions disappear [19]. Again the increasing curvature of the S^3 unit cells, and not just their reduced volume, was found to play an explicit dynamical role in achieving chiral restoration. (In addition, the S^3 description naturally generalizes to finite temperature and reveals interesting analogies between the “geometric” implementations of temperature and density [20].) The above results suggest that the S^3 unit-cell geometry encodes *chiral* interactions of nucleons with the ambient matter [19], in addition to those encoded in the flat-space model Lagrangian [28]. (The distortions of the S^3 geometry considered in Ref. [1] may thus be regarded as admixing additional interactions with the ambient matter, described by the pions’ interactions with the deformed cell background. The latter break chiral symmetry explicitly and thus prevent exact chiral restoration.)

When attempting to put the above interpretation of the cell curvature as mediating chiral interactions with the surrounding baryons on a more solid basis, the lower bound on the Skyrmon energy, which can be saturated only on S^3 , provides a valuable hint. Indeed, one may in principle determine the cell geometry at a given density variationally, as done e.g. in condensed-matter physics.

When minimizing the cell energy, including the contributions from interactions with the surroundings, one may then allow by some stretch of the imagination not just the flat, extrinsic geometry (i.e. distances and boundary conditions) but even the intrinsic curvature of the cell to vary [19]. According to the arguments of Sec. II the resulting cell geometry should then be S^3 , at least beyond the critical density. In analogy to translating interactions with surrounding baryons into the variationally determined structure of flat unit cells, the S^3 curvature would then indeed encode additional, chiral interactions of the Skyrmon with the ambient matter.

Even with such a potential dynamical origin of S^3 unit cells in mind, however, it still seems counterintuitive that their intrinsic curvature and missing boundary prevent them from being embedded into flat space. Nevertheless, such curved cells should represent identical units of a self-repeating structure which describes a finite average baryon density, to be identified with the inverse of their volume, over macroscopic distances. In order to guess a potential explanation for the above observations it seems again helpful to draw intuition from the unique capability of S^3 cells to restore chiral symmetry. The latter agrees with QCD expectations and goes far beyond the discrete half-Skyrmion (sub-) symmetry which Skyrmon arrays can restore [29]. Hence S^3 cells represent at least this crucial aspect of chiral dense-matter physics more completely than flat, periodic unit cells, and this is possible because they in a sense (cf. Sec. II) restore the translational symmetry which crystals break spontaneously.

Continuing this line of thought, it is tempting to speculate that S^3 cells cannot be embedded into \mathbb{R}^3 and directly match on to “adjacent” cells because – unlike flat, periodic cells – they do not just encode interactions with their immediate “neighbors”. More specifically, their intrinsic curvature may contain information on the averaged interactions with more distant or even all other cells. The non-locality of such averages could then reflect itself in a “dissolution” of the cell boundary. Moreover, the implied averaging procedure should be able to restore translational symmetry, which may be a prerequisite for full chiral restoration.

Rather than contemplating additional physics which may potentially be encoded in the cell curvature, the minimalistic interpretation of Ref. [1] takes the opposite route. It tentatively ignores even the impact of the extrinsic cell geometry and postulates that at least the qualitative physics should not depend on it (cf. Sec. I). Hence the role of the cell is reduced to just providing a computationally convenient volume with essentially arbitrary geometry to constrain the spreading of the Skyrmon. In view of the S^3 geometry’s unique impact discussed in Sec. II, this assumption cannot be even qualitatively correct. Instead, the cell geometry (and topology) matters even at the qualitative level, as it does in the flat unit cells of conventional Skyrmon crystals with their remarkable sensitivity to the boundary conditions. (In fact, without the latter the half-Skyrmion symmetry would not

emerge.) On S^3 the impact of the geometry is further enhanced by the heightened sensitivity of the chiral dynamics to the background curvature [30]. (Since the unit cell's boundary conditions determine the crystal structure up to scales, their neglect would be inadequate in condensed-matter physics as well, incidentally.)

Finally, it is instructive to reflect upon the impact of Ref. [1]'s main result on the interpretation of S^3 cells. The authors of Ref. [1] argue that all spatially-averaged chiral order parameters in flat-space Skyrme models and large- N_c QCD can simultaneously vanish only if chiral symmetry is also restored in the conventional, local sense (as signaled by the vanishing quark condensate in QCD). Since at least naively the former seems to happen without the latter in S^3 cells, one may suspect a contradiction with the hypersphere description of dense matter [31]. As a matter of fact, the problematic interpretation of Ref. [1] creates such a contradiction by postulating that S^3 cells should faithfully model the flat unit cells of Skyrminion crystals.

In the interpretation of S^3 cells as an independent description, on the other hand, it may at first appear that averting a contradiction requires some of the physics encoded in hyperspherical cells to differ from that of dense matter in Skyrme models and large- N_c QCD. Given the simplicity of the S^3 cell description and the lack of a first-principles derivation, this conclusion would not even be surprising. It is important to realize, however, that it would also be premature. This is because spatial averaging over the S^3 cell does not have to translate into uniform spatial averaging over some flat-space configurations, including those which the S^3 cells are supposed to describe [32]. In the tentative interpretation of the previous paragraphs this becomes particularly obvious because in such a scenario the volume of a particular S^3 cell does not even correspond to a specific and exclusive

volume of dense matter in flat space.

Hence one should keep in mind that chiral restoration “in the *flat-space* average sense” as considered in Ref. [1] is not identical to chiral restoration “in the curved-cell average” as it occurs in S^3 cells. As a consequence, there is *a priori* no reason for conclusions regarding chiral restoration in the flat-space average sense, including those of Ref. [1], to apply to chiral restoration in S^3 cells as well. In fact, as alluded to above one may optimistically hope that the latter describes a situation which corresponds more closely to the conventional, i.e. local chiral restoration in dense matter than to the flat-space averaged version dealt with in Ref. [1].

IV. CONCLUSION

In this note we clarify that the interpretation of the hypersphere approach to dense matter as tentatively considered in Ref. [1] is based on inadequate premises. Although not always obvious in its presentation, the criticism of Ref. [1] does therefore not apply to the standard interpretation and its chiral-restoration mechanism as described above. In particular, the standard interpretation does not require hyperspherical cells to be models of flat unit cells (several analogies and shared features notwithstanding), and it ascribes specific physical significance to the cell geometry as encoding chiral interactions with the ambient matter. We furthermore point out differences between chiral restoration “in the spatial-average sense” in flat space and in curved cells, and we provide a few additional suggestions concerning the dynamical origin and potential physics content of S^3 cells.

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- [1] P. Adhikari, T.D. Cohen, R.R.M. Ayyagari and M.C. Strother, Phys. Rev. C **83**, 065201 (2011).
 - [2] N.S. Manton and P. Ruback, Phys. Lett. B **181**, 137 (1986).
 - [3] N.S. Manton, Commun. Math. Phys. **111**, 469 (1987).
 - [4] H. Forkel, A.D. Jackson, M. Rho, C. Weiss, A. Wirzba and H. Bang, Nucl. Phys. A **504**, 818 (1989).
 - [5] I. Zahed and G.E. Brown, Phys. Rep. **142**, 1 (1986).
 - [6] N. S. Manton, Phys. Lett. B **192**, 177 (1987); E. Wuest, G. E. Brown and A. D. Jackson, Nucl. Phys. A **468**, 450 (1987); A. S. Goldhaber and N. S. Manton, Phys. Lett. B **198**, 231 (1987).
 - [7] I. R. Klebanov, Nucl. Phys. B **262** (1985) 133; M. Kugler and S. Shtrikman, Phys. Lett. B **208**, 491 (1988); A.D. Jackson, J.J.M. Verbaarschot, Nucl. Phys. A **484**, 419 (1988); L. Castillejo, P.S.J. Jones, A.D. Jackson, J.J.M. Verbaarschot and A. Jackson, Nucl. Phys. A **501**, 801 (1989); M. Kugler and S. Shtrikman, Phys. Rev. D **40**, 3421 (1989). For more recent work see e.g. B.-Y. Park and V. Vento, arXiv:0906.3263 [hep-ph]; J. Silva Lobo and R.S. Ward, Phys. Lett. B **696**, 283 (2011).
 - [8] L. Bratek, Phys. Rev. D **78**, 025019 (2008).
 - [9] L. McLerran and R. D. Pisarski, Nucl. Phys. A **796**, 83 (2007); Y. Hidaka, L. D. McLerran and R.D. Pisarski, Nucl. Phys. A **808**, 117 (2008).
 - [10] S.R. Coleman, J. Wess and B. Zumino, Phys. Rev. **177**, 2239 (1969); C.G. Callan, Jr., S.R. Coleman, J. Wess and B. Zumino, Phys. Rev. **177** 2247 (1969).
 - [11] G.S. Adkins, C.R. Nappi and E. Witten, Nucl. Phys. B **228**, 552 (1983).
 - [12] H. Forkel, A.D. Jackson, M. Rho and N. Scoccola, Nucl. Phys. A **509**, 673 (1990).
 - [13] J.H. Kim, S. Yee and H.K. Lee, Phys. Rev. D **53**, 1715 (1996); S. Yee, J.H. Kim, H.K. Lee, J. Korean Phys. Soc. **28**, 576 (1995); S.-T. Hong, Phys. Lett. B **417**, 211 (1998); T. Sakai and H. Suganuma, Phys. Lett. B **430**, 168 (1998); S. Krusch, Nonlinearity **13**, 2163 (2000); S.W. Goatham and S. Krusch, J. Phys. A **43**, 035402 (2010).
 - [14] K.-Y. Kim, S.-J. Sin and I. Zahed, JHEP **09**, 001 (2008).

- [15] H. Forkel, arXiv:hep-th/0407166.
- [16] T.M. Samols, preprint DAMTP/89-16; N.S. Manton and T.M. Samols, J. Phys. A **23**, 3749 (1990).
- [17] Y. Nambu and G. Jona-Lasinio, Phys. Rev. **122**, 345 (1961); **124**, 246 (1961).
- [18] H. Forkel, Phys. Lett. B **280**, 5 (1992).
- [19] H. Forkel, Nucl. Phys. A **581**, 557 (1995).
- [20] H. Forkel and A.D. Jackson, arXiv/hep-ph:9501006.
- [21] C. Weiss and A.D. Jackson, Nucl. Phys. A **547**, 551 (1992).
- [22] H.-J. Lee, B.-Y. Park, D.-P. Min, M. Rho and V. Vento, Nucl. Phys. A **723**, 427, (2003).
- [23] H.-J. Lee, B.-Y. Park, M. Rho and V. Vento, Nucl. Phys. A **726**, 69, (2003); B.-Y. Park, M. Rho and V. Vento, Nucl. Phys. A **807**, 28 (2008).
- [24] Obviously, computational simplicity is a welcome side benefit which makes analytical studies possible, in contrast to crystal calculations (beyond the Wigner-Seitz approximation) which require numerical computations. Nevertheless, this is not a major reason for preferring S^3 (although this impression has sometimes been given in the literature). In fact, computations on several other, similarly symmetric compact manifolds are not more involved (cf. Refs. [3] and [8] for explicit examples).
- [25] A similar view may have occasionally been implied in the earlier literature on the subject.
- [26] This may be of some independent use because the foundations of the hypersphere approach were occasionally simplified in the literature.
- [27] We recall that the identity map between S^3 's of *different* radii is not always the map of lowest energy. In fact, the identity map is stable only for radii L of $\Sigma_{\text{cell}} = S^3(L)$ smaller than the critical radius at which the bound is saturated and the chiral restoration transition takes place [3]. For larger radii, in contrast, the Skyrmion starts to localize on S^3 as on \mathbb{R}^3 .
- [28] Problems encountered with MIT bags in the S^3 unit cell [21] may therefore be related to the chiral-symmetry breaking bag boundary.
- [29] Higher-dimensional order parameters [1, 4] and a dense-matter generalization of the pion decay constant [22] do not vanish, for example, when only the half-Skyrmion symmetry is restored. (A more genuine chiral restoration was proposed to occur in Skyrmion arrays when the expectation value of an additional dilaton field vanishes [23].)
- [30] As a case in point, several different geometries were found to generate qualitatively different physics in the same cell volume [3, 8].
- [31] One could try to evade such a contradiction from the outset, of course, by tentatively assuming that S^3 cells incorporate relevant $1/N_c$ corrections and thereby generate a phase diagram which is closer to QCD (with $N_c = 3$) expectations.
- [32] In fact, even in the (not viable) interpretation of S^3 cells as faithful models of flat unit cells it would not be immediately clear whether and how a pointwise correspondence between the cell volumes could be established.